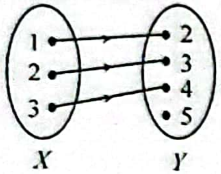
ONE PAGE NOTES
"FUNCTIONS"

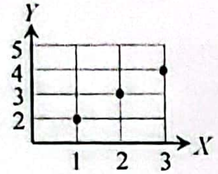
$f^n(x) = x \rightarrow f^{kn}(x) = x$

Relations representation : arrow diagram, ordered pairs, cartesian graph

arrow diagram | ordered pairs | cartesian graph



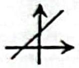
$X = \{1, 2, 3\}$
$Y = \{2, 3, 4, 5\}$
$\{(1, 2), (2, 3), (3, 4)\}$



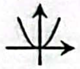
- domain = { 1, 2, 3 } , codomain = { 2, 3, 4, 5 }
- range= { 2, 3, 4 } → range ⊂ codomain
- the image of 1 = 2 , the image of 2 = 3, the image of 3 = 4
- the object of 2 = 1, the object of 3 = 2, the object of 4 = 3

Types of relation

one to one



many to one



one to many

many to many

Functions

- *function notation* → $f(x) = x + 1$ (example : refer arrow diagram)
- *conditions function* → one to one @ many to one relation
  → each object must mapped with an image only
- *to determine whether a graph is a function :*
  ~ draw a *vertical line*, and it cut the graph at only one point
- *maps onto itself :*
  ~ if x is maps onto itself under function h → $h(x) = x$
  ~ if 5 is maps onto itself under function k → $k(5) = 5$
- *defination :* $f(x) = \dfrac{ax+b}{cx+d}$, $x \neq k$
  ~ if $f(x)$ is undefined → $ck + d = 0$ @ $k = -\frac{d}{c}$
  ~ if $f(x)$ is defined → $ck + d \neq 0$ @ $k \neq -\frac{d}{c}$

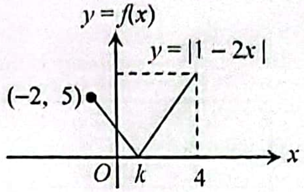
Functions : object and image

Given $f(x) = 3x - 7$. Find

(a) the image of 5, (b) the object of 23.

$f(5) = 3(5) - 7$ | $3x - 7 = 23$
$= 8$ | $x = 10$

Obsolute functions ~ object, image, domain, range



Find

(a) the value of k,
$f(k) = 0$
$|1 - 2k| = 0$
$k = \frac{1}{2}$

(b) the range of $f(x)$ for the domain $-2 \leq x \leq 4$,
$f(4) = 7 \rightarrow 0 \leq f(x) \leq 7$

(c) the domain of $0 \leq f(x) \leq 5$.
$f(x) = 5$
$|1 - 2x| = 5$
$1 - 2x = -5,\ 1 - 2x = 5$
$x = 3,\ -2 \rightarrow -2 \leq x \leq 3$

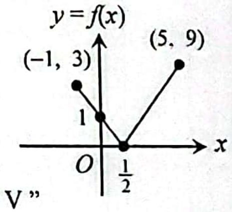
Note : $|x| = t$ ↓ $x = -t,\ t$

Steps to draw obsolute function graph

(1) find the x-intercept, when image = 0
(2) find the y-intercept, when object = 0
(3) draw the graph for the given domain

example : $f(x) = |2x - 1|$, for $-1 \leq x \leq 5$

x	0	$\frac{1}{2}$	−1	5
y	1	0	3	9



Note : graph in the shape of " V "

Composite functions : ~ *find function in front*

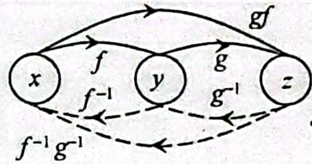
$g(x) = 3 - x$ and $fg(x) = 3x + 1$, find $f(x)$.

$f[g(x)] = 3x + 1$
$f(3 - x) = 3x + 1$
$3 - x = y$
$3 - y = x$
$f(y) = 3(3 - y) + 1$
$= 10 - 3y$
$f(x) = 10 - 3x$

@

$g^{-1}(x) = \frac{x-3}{-1} = 3 - x$
$fg[g^{-1}(x)] = 3[g^{-1}(x)] + 1$
$f(x) = 3(3 - x) + 1$
$= 10 - 3x$

$ff^{-1}(x) = f^{-1}f(x) = x$
↓
$ff^{-1}[k(x)] = k(x)$



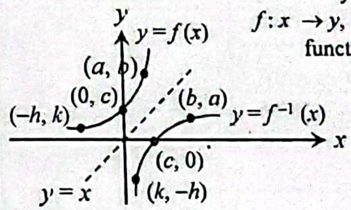
$f(x) = y;\ g(y) = z \rightarrow gf(x) = z$

$g^{-1}(z) = y;\ f^{-1}(y) = x \rightarrow f^{-1}g^{-1}(z) = x$

function $g \bullet f$ exists → range of $f \subseteq$ domain of g
function $f \bullet g$ exists → range of $g \subseteq$ domain of f

Inverse functions

- to every *one to one* function $f : x \rightarrow y$, there exist an inverse function $f^{-1} : y \rightarrow x$



- domain for $f^{-1}(x)$ = range for $f(x)$
  range for $f^{-1}(x)$ = domain for $f(x)$
- the graph of $y = f(x)$ and $y = f^{-1}(x)$ are symmetry about the line $y = x$
- if $y = f(x)$ and $y = f^{-1}(x)$ are inverse to each other → $ff^{-1}(x) = f^{-1}f(x) = x$
- *determine whether a function has an inverse :* draw a *horizontal line*, and it cut the graph at only one point

$f(x) = x + k \rightarrow f^{-1}(x) = x - k$
$f(x) = kx \rightarrow f^{-1}(x) = \frac{x}{k}$
$f(x) = ax + b \rightarrow f^{-1}(x) = \frac{x-b}{a}$
$f(x) = \frac{x}{a} + b \rightarrow f^{-1}(x) = a(x - b)$
$f(x) = x^2 \rightarrow f^{-1}(x) = \sqrt{x}$

$g(x) = \frac{3x}{x-4},\ x \neq 4$
$\frac{3x}{x-4} = y$
$3x = xy - 4y$
$4y = xy - 3x$
$4y = x(y - 3)$
$\frac{4y}{y-3} = x$
$g^{-1}(x) = \frac{4x}{x-3},\ x \neq 3$

Composite functions :

$u(x) = 1 + 3x$,
$v(x) = 3 - 2x$.

Find :

(a) $vu(x)$,
$= v[u(x)]$
$= v(1 + 3x)$
$= 3 - 2(1 + 3x)$
$= 1 - 6x$

(b) $u^2(2)$.
$= u[u(2)]$
$= u[1 + 3(2)]$
$= u(7)$
$= 22$

~ *find function in behind*

$f(x) = 2x - 5$, and $fg(x) = 1 - 2x$, find

(a) $fg(-1)$,
$= 1 - 2(-1)$
$= 3$

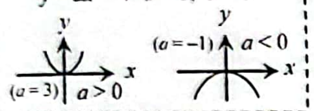
(b) $g(x)$.
$f[g(x)] = 1 - 2x$,
$2[g(x)] - 5 = 1 - 2x$
$2[g(x)] = 6 - 2x$
$g(x)] = 3 - 2x$

$f(x) = \frac{ax+b}{cx+d} \rightarrow f^{-1}(x) = \frac{dx-b}{-cx+a}$

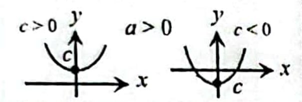
(for checking answer)

ONE PAGE NOTES
" QUADRATIC FUNCTIONS "

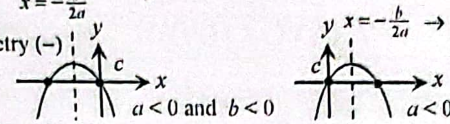
$y = ax^2 \rightarrow b = 0,\ c = 0$



$y = ax^2 + c \rightarrow b = 0$



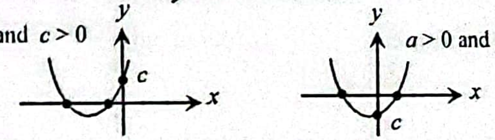
$y = ax^2 + bx \rightarrow c = 0$



If a and b are same sign → symmetry to the *left* of y-axis

If a and b are different sign → symmetry to the *right* of y-axis

$y = ax^2 + bx + c$



If a and c are same sign → roots are same sign → $ac > 0$

If a and c are different sign → roots are different sign → $ac < 0$

Quadratic Equations $ax^2 + bx + c = 0$ ← $f(x) = 0$ ← *Quadratic Functions* $f(x) = ax^2 + bx + c$ → *Quadratic Inequalities*

find the roots of the quadratic equation (the value of x that satisfies the equation)

using calculator

(*CASIO fx*-570MS, *CANON F*-570SG)

[mode] [mode] [mode]

[1] [▸] [2]

$a =$ $b =$ $c =$

$\rightarrow x_1 = ?$ $x_2 = ?$

NOTE : if appear x only

$\rightarrow x_1 = x_2$

using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

completing the square

$x^2 + bx + c = 0$

$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$

$(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c = 0$

@

$x^2 - bx + c = 0$

$x^2 - bx + \left(\frac{-b}{2}\right)^2 - \left(\frac{-b}{2}\right)^2 + c = 0$

$(x - \frac{b}{2})^2 - (-\frac{b}{2})^2 + c = 0$

@

$ax^2 + bx + c = 0$

$a[x^2 + \frac{b}{a}x] + c = 0$

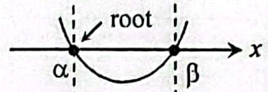
$a[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2] + c = 0$

$a[(x + \frac{b}{2a})^2 - (\frac{b}{2a})^2] + c = 0$

$a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c = 0$

Discriminant → $b^2 - 4ac$

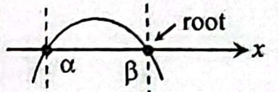
$y = f(x) \rightarrow a > 0$



$f(x) < 0 \rightarrow \alpha < x < \beta$

$f(x) > 0 \rightarrow x < \alpha,\ x > \beta$

$y = f(x) \rightarrow a < 0$



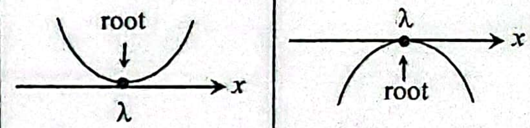
$f(x) \geq 0 \rightarrow \alpha \leq x \leq \beta$

$f(x) \leq 0 \rightarrow x \leq \alpha,\ x \geq \beta$

$f(x)$ intersect x-axis at 2 distinct points

[2 different / distinct real roots]

$b^2 - 4ac > 0$ *(positive discriminat)*



$f(x) = (x - \lambda)^2$ $f(x) = -(x - \lambda)^2$

$f(x)$ touches x-axis at only 1 point

x-axis is the tangent to $f(x)$

[2 equal real roots]

$b^2 - 4ac = 0$

Vertex Form

$f(x) = a(x - h)^2 + k$

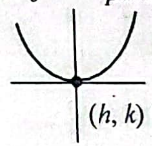
- is reflected at x-axis

  $f(x) = -a(x - h)^2 - k$

- is reflected at y-axis

  $f(x) = a(x + h)^2 - k$

$a > 0$ *parabola*



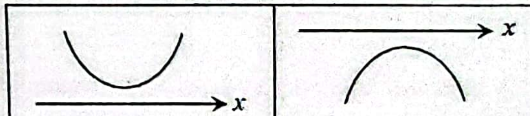
[equation of tangent]

$y = k$

(h, k) [minimum point]

$x = h$

[equation of axis of symmetry]



$f(x)$ does not intersect x-axis

$f(x)$ is always above @ below x-axis

$f(x)$ is always positive @ negative

[no real roots / imaginary roots]

$b^2 - 4ac < 0$ *(negative)*

General Form

$f(x) = ax^2 + bx + c$

↓

$a < 0$, maximum point

$\left[-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right]$

↓

$f\left(-\frac{b}{2a}\right) = \frac{4ac - b^2}{4a}$

Intercept Form

$f(x) = a(x - p)(x - q)$

↓

Turining point / vertex

$\left[\frac{p+q}{2},\ f\left(\frac{p+q}{2}\right)\right]$

$k = f\left(-\frac{b}{2a}\right) = \frac{4ac - b^2}{4a} = f\left(\frac{p+q}{2}\right)$

$h = -\left(\frac{b}{2a}\right) = \frac{p+q}{2}$

REMARKS : real roots $b^2 - 4ac \geq 0$

Form Quadratic Equations from given Roots

roots $= \alpha,\ \beta$

$(x - \alpha)(x - \beta) = 0$

@

$x^2 - (SOR)x + (POR) = 0$ ~ $SOR = \alpha + \beta$ & $POR = \alpha\beta$

IDENTITIES ALGEBRAIC

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$SOR = -\left(\frac{b}{a}\right)$ and $POR = \frac{c}{a}$

⇒ ***SOR*** = sum of roots & ***POR*** = product of roots

domain : $-\infty \leq x \leq \infty$

relation ~ many to one

domain : $x \leq h$ @ $x \geq h$

relation ~ one to one

Sketch the graph of a quadratic function

(i) find the minimum @ maximum point

(ii) find the y-intercept

→ value of c @ substitute $x = 0$, $f(0)$

(iii) find the x-intercept

→ substitute $y = 0$ ~ calculator (*EQN*)

(iv) find the value of $f(m)$ and $f(n)$ for the given domain : $m \leq x \leq n$.

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STEPS OF SOLUTION : SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES

~ each linear equation has 3 variables
~ the highest power of each variable is 1

- *steps in solution : using elimination method*

(1) Rearrange all the equations in the form, $ax + by + cz = d$.
(2) Do the first elimination, either x, y or z
→ *NOTE* : same sign (−), different sign (+)
(3) Do the second elimination. ~ obtained the value of the first variable.
(4) Substitute the value of first variable into any equations in (2) to get the value of second variable.
(5) Substitute the values of first and second variable into any equations in (1) to get the value of third variable.

example 1 : *linear equation in 3 variables*

Solve the system of linear equations :

$x - 3y + z = 2$ ..... (1)
$4x - 4y + z = 7$ ..... (2)
$2x + y - 3z = -4$ ...... (3)

(1) × 4 ; (2)

$4x - 12y + 4z = 8$
−) $4x - 4y + z = 7$
$-8y + 3z = 1$ ... (4)

(1) × 2 ; (3)

$2x - 6y + 2z = 4$
−) $2x + y - 3z = -4$
$-7y + 5z = 8$ ... (5)

(4) × 7 ; (5) × 7

$-56y + 21z = 7$
−) $-56y + 40z = 64$
$-19z = -57$
$z = 3$ → (4)

- $-8y + 9 = 1$
  $y = 1$
- $y = 1$, $z = 3$ → (1)
  ~ $x - 3 + 3 = 2$
  $x = 2$

example 2 :

Solve the system of linear equations :

$y - 7z = -2$ ..... (1)
$x - y + 5z = 2$ ..... (2)
$-2x + 2y - 10z = 6$ ...... (3)

(2) × 2 ; (3)

$2x - 2y + 10z = 4$
+) $-2x + 2y - 10z = 6$
$0 = 10$
~ $0 \neq 10$ → no solution

if $0 = 0$ ↓ infinite solutions

example 1 : *simultaneous equations ~ calculator / factor*

Solve the simultaneous equations :

$2x - y - 3 = 0$ and $2x^2 + y = 10x - 9$

- $2x - y - 3 = 0$
  → $2x - 3 = y$
- $2x^2 + y = 10x - 9$ ←
  $2x^2 + 2x - 3 - 10x + 9 = 0$
  $2x^2 - 8x + 6 = 0$
  $x^2 - 4x + 3 = 0$
  $(x - 1)(x - 3) = 0$
  $x = 1$, $x = 3$
- $x = 1$, $y = -1$
  $x = 3$, $y = 3$

example 2 : *simultaneous equations ~ formula*

Solve the simultaneous equations :

$2x + y = 1$ and $2x^2 + y^2 + xy = 5$

Give your answers correct to three decimal places.

- $2x + y = 1$
  → $y = 1 - 2x$
- $2x^2 + y^2 + xy = 5$ ←
  $2x^2 + (1 - 2x)^2 + x(1 - 2x) - 5 = 0$
  $2x^2 + 1 - 4x + 4x^2 + x - 2x^2 - 5 = 0$
  $4x^2 - 3x - 4 = 0$
- $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-4)}}{2(4)}$
  $= \dfrac{3 \pm \sqrt{73}}{8}$
  $= 1.443, \ -0.693$
- $x = 1.443$, $y = -1.886$
  $x = -0.693$, $y = 2.386$

$(a + b)^2 = a^2 + 2ab + b^2$
→ $(x + 7)^2 = x^2 + 14x + 49$

$(a - b)^2 = a^2 + 2ab + b^2$
→ $(2x - 1)^2 = 4x^2 - 4x + 1$

ONE PAGE NOTES
" SYSTEMS OF EQUATIONS "

STEPS OF SOLUTION : SIMULTANEOUS EQUATIONS

- *steps in solution : using subtituition method*

(1) From the linear equation, express one of the unknown as the subject. ~ sometime, also can from the non-linear equation
(2) Suistituted (1) into the non-linear equation to form a quadratic equation.
(3) Arranged the quadratic equation into general form : $ax^2 + bx + c = 0$
(4) Simplify and solve the quadratic equation by using :

(i) *calculator fx-570MS*

$3x^2 - 5x - 12 = 0$
$(x - 3)(3x + 4) = 0$ ⇐ MUST
$x = 3$ $x = -\frac{4}{3}$

from calculator : fx-570MS

[mode] [mode] [mode] [1] [▸] [2]

a ? 3 = b ? −5 = c ? −12 =

→ $x_1 = 3$ → $x_2 = -1.33...$ [shift] [a b/c] $-\frac{4}{3}$

Note : if appear x only → $x_1 = x_2$

(ii) *factorise*

$2x^2 - 5x = 0$
$x(2x - 5) = 0$
$x = 0$ $x = \frac{5}{2}$

(iii) *formula* $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(5) Obtain the values of other unknows by substituting the x_1 and x_2 into (1).

$(a + b)(a - b) = a^2 - b^2$
→ $(3x + 2)(3x - 2) = 9x^2 - 4$

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" INDICES, SURDS AND LOGARITHMS "

LAWS OF INDICES

$a^m \times a^n = a^{m+n}$	$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{m\times n} = (a^n)^m$	$a^0 = 1 \rightarrow a \neq 0$
$a^n \times b^n = (a \times b)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$	$\frac{1}{a^n} = a^{-n}$ @ $\frac{1}{a^{-n}} = a^n$	
$\frac{1}{a^{m-n}} = a^{-m+n}$	$\frac{k}{a^{-m+n}} = k a^{m-n}$	$\frac{1}{a^m \times a^n} = a^{-(m+n)}$ @ $\frac{1}{a^m \div a^n} = a^{-(m-n)}$	
$\sqrt[n]{a^m} = a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$		$\sqrt[n]{a} = a^{\frac{1}{n}} \Rightarrow \sqrt{a} = a^{\frac{1}{2}}$ & $\sqrt[3]{a} = a^{\frac{1}{3}}$	

Notes : $a^n = y \Leftrightarrow a = y^{\frac{1}{n}}$ @ $a^{\frac{m}{n}} = y \Leftrightarrow a = y^{\frac{n}{m}}$

TABLE FOR NUMBERS POWER OF n

BASE 2	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
BASE 3	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
BASE 5	$5^{-3} = \frac{1}{125}$	$5^{-2} = \frac{1}{25}$	$5^{-1} = \frac{1}{5}$	$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	
BASE 6	$6^{-3} = \frac{1}{216}$	$6^{-2} = \frac{1}{36}$	$6^{-1} = \frac{1}{6}$	$6^0 = 1$	$6^1 = 6$	$6^2 = 36$	$6^3 = 216$		$2^6 = 64$
BASE 7	$7^{-3} = \frac{1}{343}$	$7^{-2} = \frac{1}{49}$	$7^{-1} = \frac{1}{7}$	$7^0 = 1$	$7^1 = 7$	$7^2 = 49$	$7^3 = 343$	$2^{10} = 1024$	$2^7 = 128$
BASE 10	$10^{-3} = \frac{1}{1000}$	$10^{-2} = \frac{1}{100}$	$10^{-1} = \frac{1}{10}$	$10^0 = 1$	$10^1 = 10$	$10^2 = 100$	$10^3 = 1000$	$2^9 = 512$	$2^8 = 256$

Notes :

$2 = 4^{\frac{1}{2}}$	$3 = 9^{\frac{1}{2}}$	$5 = 25^{\frac{1}{2}}$	$7 = 49^{\frac{1}{2}}$	$2 = 8^{\frac{1}{3}}$	$3 = 27^{\frac{1}{3}}$

SURDS → numbers with radicals, have *infinite decimal places* and are *non-recurring / not a fraction*

$\sqrt{a} \times \sqrt{a} = a$	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$	$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	$\left(\sqrt{a} \pm \sqrt{b}\right)^2 = a + b \pm 2\sqrt{a}\sqrt{b}$	$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a - b$

$\sqrt{4} = 2$	$\sqrt{9} = 3$	$\sqrt{16} = 4$	$\sqrt{8} = 2\sqrt{2}$	$\sqrt{32} = 4\sqrt{2}$	$\sqrt{50} = 5\sqrt{2}$	$\sqrt{72} = 6\sqrt{2}$	$\sqrt{20} = 2\sqrt{5}$
$\sqrt{25} = 5$	$\sqrt{36} = 6$	$\sqrt{49} = 7$	$\sqrt{12} = 2\sqrt{3}$	$\sqrt{27} = 3\sqrt{3}$	$\sqrt{147} = 7\sqrt{3}$	$\sqrt{192} = 8\sqrt{3}$	$\sqrt{45} = 3\sqrt{5}$

$\log_a x^n = n \log_a x$	$\log_{a^n} x = \frac{\log_a x}{n} = \log_a x^{\frac{1}{n}}$

LOGARITHMS AND LAWS OF LOGARITHMS (1)

$a^x = y$ ($y > 0$) ⟵ - - - ⟶ $x = \log_a y$ ($y > 0$; $a > 0,\ a \neq 1$)	$a^{\log_a y} = y$
	$\log_a 1 = 0$
$\log_a b = \frac{\log_c b}{\log_c a}$; $\log_a b = \frac{1}{\log_b a}$	$\log_a a = 1$
$\log_a b = k \Leftrightarrow \log_b a = \frac{1}{k}$	$\lg y = \log_{10} y$
$\log_a (xy) = \log_a x + \log_a y$	$\log_a \left[\frac{x}{y}\right] = \log_a x - \log_a y$

LOGARITHM OF A NUMBER

$-2 = \log_2 \frac{1}{4} = \log_3 \frac{1}{9} = \log_5 \frac{1}{25} = \ldots = \log_x x^{-2}$

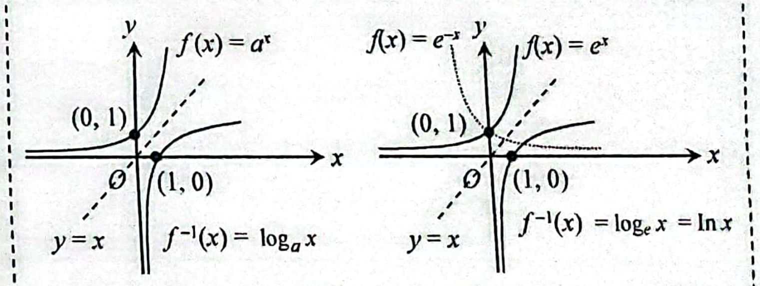
$-1 = \log_2 \frac{1}{2} = \log_3 \frac{1}{3} = \log_5 \frac{1}{5} = \ldots = \log_x x^{-1}$

$0 = \log_2 1 = \log_3 1 = \log_5 1 = \ldots = \log_x 1$

$1 = \log_2 2 = \log_3 3 = \log_5 5 = \ldots = \log_x x$

$2 = \log_2 4 = \log_3 9 = \log_5 25 = \ldots = \log_x x^2$

$3 = \log_2 8 = \log_3 27 = \log_5 125 = \ldots = \log_x x^3$



NOTE : natural logarithm, $\log_e$ @ ln

additional notes :

- $\left[\sqrt{a}\right]^3 = a\sqrt{a}$
- $\log_{\sqrt{x}} 7 = \log_x 7^2$
- $\frac{1}{\log_x xy} + \frac{1}{\log_y xy} = \log_{xy} x + \log_{xy} y$

rationalising the denominator

$\sim p\sqrt{a} - q\sqrt{b}$ ⟸ conjugate surd ⟹ $p\sqrt{a} + q\sqrt{b}$

HINT

$\ln e = 1$; $\ln e^x = x$; $e^{\ln x} = x$

$e^x = y \Leftrightarrow x = \ln y$

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ONE PAGE NOTES "PROGRESSIONS"

FOR : Arithmetic Progression & Geometric Progression

$$\overbrace{\underbrace{T_1 + T_2 + T_3}_{S_3} + \underbrace{T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15}}_{S_{15} - S_3}}^{S_{15}}$$

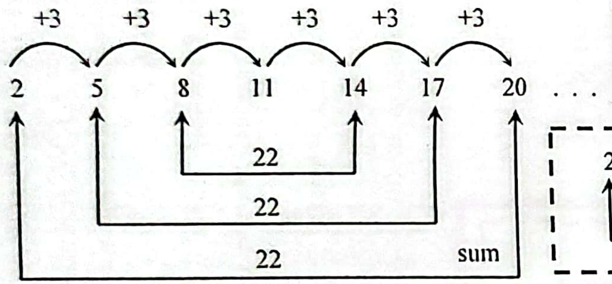
$T_1 = S_1 = a$
$T_2 = S_2 - S_1$
$T_3 = S_3 - S_2$
⋮
$T_n = S_n - S_{n-1}$

ARITHMETIC PROGRESSION

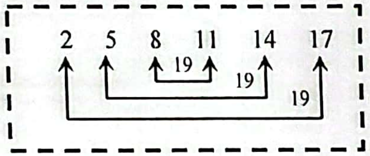
to derive the formula of S_n:

$$\begin{aligned} S_n &= a + \ldots + T_n \\ +)\quad S_n &= T_n + \ldots + a \\ \hline 2S_n &= n(a + T_n) \end{aligned}$$

(i) characteristics



$a = 2$, first term
$d = 3$, common difference



$\therefore\ d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \ldots = T_n - T_{n-1} \rightarrow d \neq 0$; $n > 1$ @ $n \geq 2$

→ positive integers

(ii) *n-term*, T_n

$T_n = a + (n-1)d$

$T_1 = a$
$T_2 = a + d$
$T_3 = a + 2d$
$T_4 = a + 3d$
⋮

Arithmetic Mean

$$T_n = \frac{T_{n-1} + T_{n+1}}{2}$$

(iii) sum of the first n terms, S_n

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$S_1 = a$
$S_2 = \frac{2}{2}[2a + d]$
$S_3 = \frac{3}{2}[2a + 2d]$
$S_4 = \frac{4}{2}[2a + 3d]$
⋮

$S_n = \frac{n}{2}(a + T_n)$

a → First term

T_n → n- term @ last term

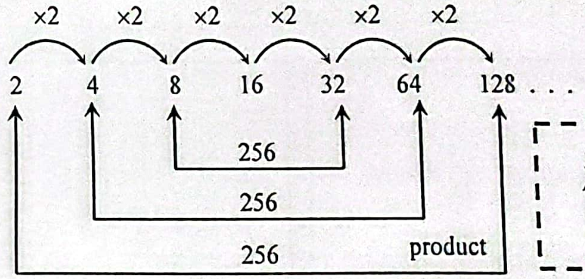
GEOMETRIC PROGRESSION

to get the formula of S_n:

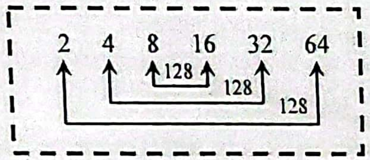
$$\begin{aligned} S_n &= a + ar + \ldots + ar^{n-1} \\ -)\quad rS_n &= ar + \ldots + ar^{n-1} + ar^n \\ \hline (1 - r)S_n &= a - ar^n \end{aligned}$$

@ rS_n −) S_n

(i) characteristics



$a = 2$, first term
$r = 2$, common ratio



$\therefore\ r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \ldots = \frac{T_n}{T_{n-1}} \rightarrow r \neq 1$; $n > 1$ @ $n \geq 2$

→ positive integers

(ii) *n-term*, T_n

$T_n = ar^{n-1}$

$T_1 = a$
$T_2 = ar$
$T_3 = ar^2$
$T_4 = ar^3$
⋮

Geometric Mean

$$(T_n)^2 = (T_{n-1})(T_{n+1})$$

(iii) sum of the first n terms, S_n

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad |r| > 1$$

@

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad |r| < 1$$

recurring @ repeating decimal

sum to infinity

$$S_\infty = \frac{a}{1 - r}, \quad \text{for } |r| < 1$$

$-1 < r < 1$

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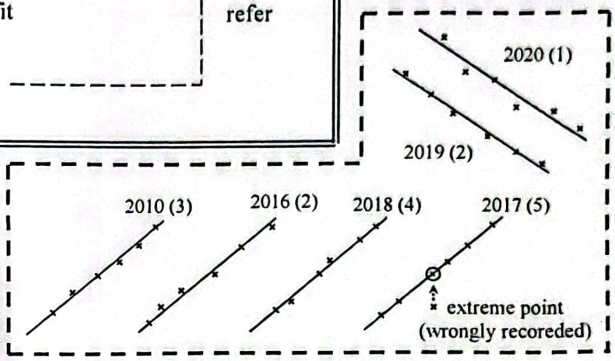
NOTA ONE PAGE ONE PAGE NOTES (Coding Method)
" LINERA LAW "

STEPS OF SOLUTION → PAPER 2

- construct table, based on the given X-axis and Y-axis.
- plot graph Y against X, based on given scale. ← refer
- draw the line of best fit. L (a)
  - ⇒ find m ~ choose two points on the line of best fit
  - ⇒ find c ~ y-intercept of the line of best fit
- reduce non linear equation to linear equation. L (b)
  - ⇒ determine m
  - ⇒ determine c
- compare the m and c for (a) an (b) & solve.
- from the graph :
  - ⇒ given value of x ~ find the value of y
  - ⇒ given value of y ~ find the value of x

gradinet, m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



REDUCE "NON-LINEAR" TO "LINEAR" → $Y = mX + c$

" TYPE 1 " - using "×", "÷", square, factor, rearrange ...

①

$y = px + \frac{r}{px}$ → $Y = xy,\ X = x^2$

$xy = px^2 + \frac{r}{p}$

⇒ $m = p\ ,\quad c = \frac{r}{p}$

②

$\frac{p}{y} = \frac{q}{x^2} - 1$ → $Y = \frac{1}{y},\ X = \frac{1}{x^2}$

$\frac{1}{y} = (\frac{q}{p})(\frac{1}{x^2}) - \frac{1}{p}$

⇒ $m = \frac{q}{p}\ ,\quad c = -\frac{1}{p}$

③

$\frac{x+3}{a} + \frac{y^2}{b} = 1$ → $Y = y^2,\ X = x + 3$

$\frac{y^2}{b} = -\left(\frac{x+3}{a}\right) + 1$

$y^2 = -\left(\frac{b}{a}\right)(x+3) + b$

⇒ $m = -\left(\frac{b}{a}\right)\ ,\quad c = b$

④

$x = py + xy$ → $Y = \frac{1}{y},\ X = \frac{1}{x}$

$x = y(p + x)$

$= \frac{p+x}{nx}$

$= \left(\frac{p}{n}\right)\left(\frac{1}{x}\right) + \frac{1}{n}$

$m = \frac{p}{n}\ ,\quad c = \frac{1}{n}$

⑤

$y = \frac{a}{x-b}$ → $Y = \frac{1}{y},\ X = x$

$\frac{1}{y} = \frac{x-b}{a}$

$\frac{1}{y} = \frac{1}{a}(x) - \frac{b}{a}$

⇒ $m = \frac{1}{a}\ ,\quad c = -\frac{b}{a}$

⑥

$T = 2\pi\sqrt{\frac{L}{g}}$ → $Y = T^2,\ X = L$

$T^2 = 4\pi^2\left(\frac{L}{g}\right)$

$T^2 = \frac{4\pi^2}{g}(L)$

⇒ $m = \frac{4\pi^2}{g}\ ,\quad c = 0$ (straight line passes O)

" TYPE 2 " - using ' *log* '

△1

$y = ab^x$ → $Y = \log_{10} y,\ X = x$

$\log_{10} y = \log_{10} a + \log_{10} b^x$

$\log_{10} y = \log_{10} a + (\log_{10} b)(x)$

⇒ $m = \log_{10} b\ ,\quad c = \log_{10} a$

△2

$y = pk^{x+1}$ → $Y = \log_{10} y,\ X = x + 1$

$\log_{10} y = \log_{10} p + \log_{10} k^{x+1}$

$\log_{10} y = \log_{10} p + (\log_{10} k)(x + 1)$

⇒ $m = \log_{10} k\ ,\quad c = \log_{10} p$

△3

$y = 100^{a + bx^2}$ → $Y = \log_{10} y,\ X = x^2$

$\log_{10} y = \log_{10} 100^{a + bx^2}$

$\log_{10} y = (a + bx^2)\log_{10} 100$

$\log_{10} y = 2a + 2bx^2$

⇒ $m = 2b\ ,\quad c = 2a$ ... [$\log_{10} 100 = 2$]

△4

$y = \frac{k}{h^{2x}}$ → $Y = \log_{10} y,\ X = x$

$\log_{10} y = \log_{10} k - \log_{10} h^{2x}$

$\log_{10} y = \log_{10} k - (2\log_{10} h)(x)$

⇒ $m = -2\log_{10} h\ ,\quad c = \log_{10} k$

△5

Wreite $T + 10 = \frac{k}{h^x}$ in linear form.

$\log_{10}(T + 10) = \log_{10} k - \log_{10} h^x$

$\log_{10}(T + 10) = \log_{10} k - (\log_{10} h)(x)$

⇒ $m = -\log_{10} h\ ,\quad c = \log_{10} k$

⇒ $Y = \log_{10}(T + 10)\ ,\quad X = x$

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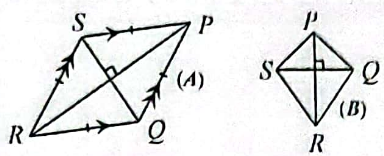
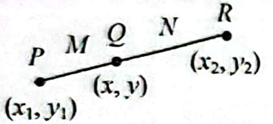
ONE PAGE NOTES
" COORDINATE GEOMETRY "



Equation of PR = equation of the perpendicular bisector of QS
(1) m_{QS}
(2) $m_{QS} \times m_{PR} = -1$
(3) titik tengah QS
(4) $y = (m_{PR})x + c$

(A) Rhombus
(B) Kite



Distance of PR $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of PR $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Line Segment

$$Q(x, y) = \left(\frac{Mx_2 + Nx_1}{M+N}, \frac{My_2 + Ny_1}{M+N}\right)$$

↓

$$\frac{Mx_2 + Nx_1}{M+N} = x \quad \& \quad \frac{My_2 + Ny_1}{M+N} = y$$

Gradient of PR, $m_{PR} = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Note :
P, Q, R are collinear → $m_{PQ} = m_{PR} = m_{QR}$
→ *area of* $\Delta PQR = 0$

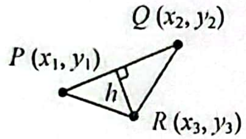
Equation of Straight Line PR
[in gradient form]

$y = mx + c$ — c → *y*-intercept; m → gradient of *PR*

Equation of Straight Line [in general form]

$ax + by + c = 0$
↓
arrange ingredient form

If point (h, k) passes through @ lies on $ax + by = c$, then the point satisfy the equation, which $ah + bk = c$

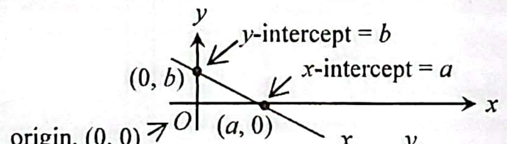


Area of $\Delta PQR = \dfrac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right| = \dfrac{1}{2}\left| (x_1)(y_2) + (x_2)(y_3) + (x_3)(y_1) - (y_1)(x_2) - (y_2)(x_3) - (y_3)(x_1) \right|$

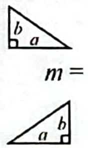
Notes : (1) point arrange in direction of '***clockwise***', the value in the obsolute will be negative, and vise-versa
(2) find shortest / perpendicular distance of R to PQ, h → $\frac{1}{2}$ × distance of $PQ \times h$ = *area of* ΔPQR

Area of quadrilateral $= \dfrac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right| = \dfrac{1}{2}\left| (x_1)(y_2) + (x_2)(y_3) + (x_3)(y_4) + (x_4)(y_1) - (y_1)(x_2) - (y_2)(x_3) - (y_3)(x_4) - (y_4)(x_1) \right|$

Area of polygon with n sides $= \dfrac{1}{2}\left|\begin{matrix} x_1 & x_2 & \cdots & x_n & x_1 \\ y_1 & y_2 & \cdots & y_n & y_1 \end{matrix}\right| = \dfrac{1}{2}\left| (x_1)(y_2) + (x_2)(y_3) + \ldots + (x_n)(y_1) - (y_1)(x_2) - (y_2)(x_3) - \ldots - (y_n)(x_1) \right|$



$$\frac{x}{a} + \frac{y}{b} = 1$$

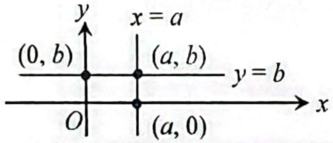


$m = -\left(\dfrac{b}{a}\right)$

$m = +\left(\dfrac{b}{a}\right)$

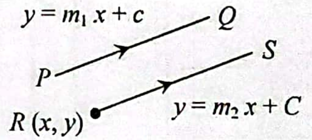
Equation of Straight Line
[***in intercept form***]
↓
$$m = -\left(\frac{b}{a}\right) = -\left(\frac{y\text{-intercept}}{x\text{-intercept}}\right)$$

Equation of Straight Lines
→ parallel to *x*-axis ~ *y*-coordinate similar ~ $y = b$
→ parallel to *y*-axis ~ *x*-coordinate similar ~ $x = a$



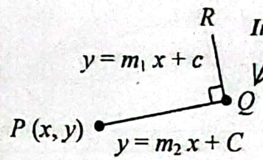
to determine whether the locus (1) @ (3) cut the x / y-intercept
→ substitute y / $x = 0$ and find the value of $b^2 - 4ac$

Concept of Parallel / Paralle Equation



$PQ \parallel RS \Rightarrow m_1 = m_2$

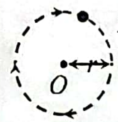
Concept of Perpendicular / Perpendicular Equation



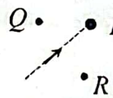
Intersection Point
PQ & *QR*
↓
substitution / elimination

$PQ \perp QR \Rightarrow m_1 \times m_2 = -1$ @ $\tan\theta_1 \times \tan\theta_2 = -1$

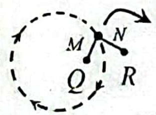
Equation of Locus ~ involving distance



$P(x, y)$ → locus of point P from 1 fixed point with a constant distance
$PO = r$ ~ a circle with centre O [*midpoint of diameter*], and radius r (1)



$P(x, y)$ → locus of point P which the distance from 2 fixed points is equal
$PQ = PR$ ~ a perpendicular bisector of QR (2)



$P(x, y)$ → locus of point P which the distance from 2 fixed points is in the ratio $M : N$
$PQ : PR = M : N$ → $N(PQ) = M(PR)$ ~ a circle (3)

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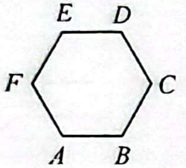
ONE PAGE NOTES

" VECTORS "

speed = | velocity vector |



$\vec{AB} = \vec{ED}$

$\vec{BC} = \vec{FE}$

$\vec{CD} = \vec{AF}$

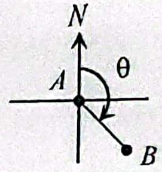
$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{CD} - \vec{ED} = \vec{CE}$

$\vec{BF} + \vec{FD} + \vec{FA} = \vec{BD} + \vec{FA}$

$= \vec{BD} + \vec{DC}$

$= \vec{BC}$



three-digit

000°

↓

360°

θ = bearing of B from A

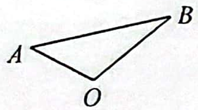
VECTORS IN CARTESIAN PLANE

$A(-3,\ 2)$

$\Rightarrow \vec{OA} = -3\,i + 2\,j = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$\Rightarrow \vec{AO} = 3\,i - 2\,j = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

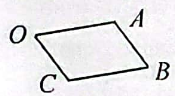
TRIANGLE LAW



$\vec{AB} = \vec{AO} + \vec{OB}$

$[\ \vec{AB} = \vec{AO} - \vec{BO}\ ]$

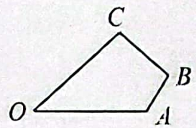
PARALLELOGRAM LAW



$\vec{OB} = \vec{OA} + \vec{AB}$

$= \vec{OC} + \vec{CB}$

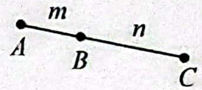
POLIGON LAW



$\vec{OC} = \vec{OA} + \vec{AB} + \vec{BC}$

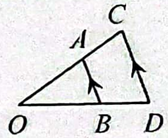
$\frac{AB}{BC} = \frac{m}{n}$

$AB : BC = m : n$



$\vec{AB} = \left(\frac{m}{m+n}\right)\vec{AC}$

$\vec{CB} = \left(\frac{n}{m+n}\right)\vec{CA}$



$\frac{AB}{CD} = \frac{OA}{OC} = \frac{OB}{OD}$

$\frac{\text{area } \Delta OAB}{\text{area } \Delta OCD} = \left(\frac{AB}{CD}\right)^2$

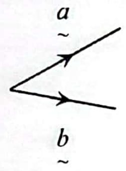
ADDITION AND SUBTRACTION OF VECTORS

$k\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$

NON- PARALLE VECTORS



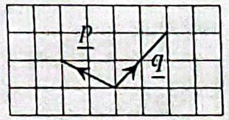
If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-parallel and non-zero, and $h\underset{\sim}{a} = k\underset{\sim}{b}$

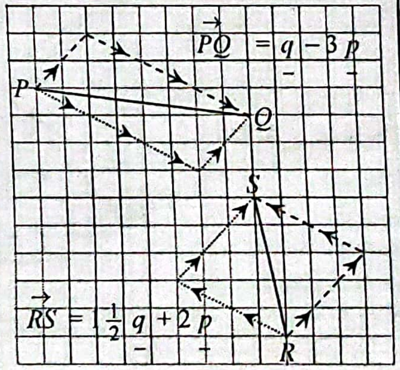
↓

$h = k = 0$

RESULTANT VECTORS

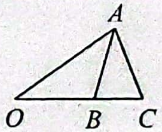
[*on a square grid*]





$\vec{PQ} = \underset{\sim}{q} - 3\underset{\sim}{p}$

$\vec{RS} = 1\frac{1}{2}\underset{\sim}{q} + 2\underset{\sim}{p}$



$\frac{\text{area } \Delta OAB}{\text{area } \Delta BAC} = \frac{OB}{BC}$

$\frac{\text{area } \Delta OAB}{\text{area } \Delta OAC} = \frac{OB}{OC}$

$\underset{\sim}{r} = x\,\underset{\sim}{i} + y\,\underset{\sim}{j} = \begin{pmatrix} x \\ y \end{pmatrix}$

MAGNITUDE @ LENGTH OF VECTORS

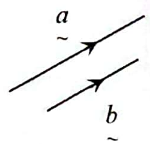
magnitude $\underset{\sim}{r} \rightarrow \left|\underset{\sim}{r}\right| = \sqrt{x^2 + y^2}$

UNIT VECTORS

unit vector $\underset{\sim}{r}$, $\underset{\sim}{\hat{r}} = \frac{\underset{\sim}{r}}{\left|\underset{\sim}{r}\right|} = \frac{1}{\left|\underset{\sim}{r}\right|}\left(\underset{\sim}{r}\right)$

Remark : $\left|\underset{\sim}{\hat{r}}\right| = 1 \rightarrow$ magnitude of unit vector = 1

PARALLE VECTORS



If $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$,

↓

$\underset{\sim}{a} = \lambda\underset{\sim}{b}$

where λ is a constant

If $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is parallel to x-axis $\Rightarrow y = 0$

If $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is parallel to y-axis $\Rightarrow x = 0$

If A, B, C are collinear $\Rightarrow AB \,//\, AC \,//\, BC$

PERPENDICULAR VECTORS

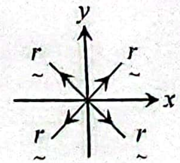
If $\underset{\sim}{a}$ is perpendicular / orthogonal to $\underset{\sim}{b}$,

↓

$(m_a)(m_b) = -1$ @ $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

VECTORS - gradient

[*translation* $\begin{pmatrix} x \\ y \end{pmatrix}$]



$\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$

$m_{\underset{\sim}{r}} = \frac{y}{x}$

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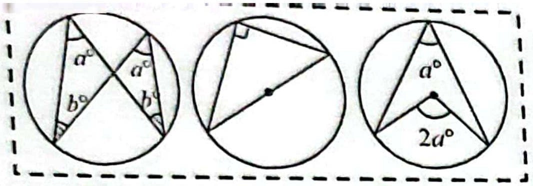
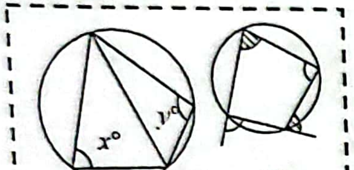
ONE PAGE NOTES (*Coding Method*)

" SOLUTION OF TRIANGLES "

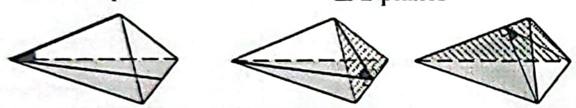
SOLUTION OF TRIANGLES





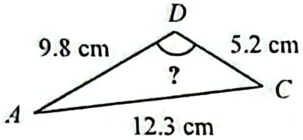
$x + y = 180$

∠ line / plane   ∠ 2 planes



" Cosine Rule "

2 — *" given 3 sides "*



$$\cos \angle ADC = \frac{9.8^2 + 5.2^2 - 12.3^2}{2(9.8)(5.2)}$$

$\angle ADC = 106.07°$

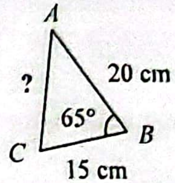
2 — area of triangle using the *"Heron's formula"*

$s = \frac{9.8 + 5.2 + 12.3}{2} = 13.65$ ~ s = semi perimeter

$area\ \Delta ADC = \sqrt{13.65\,(13.65 - 9.8)\,(13.65 - 5.2)\,(13.65 - 12.3)}$

$= 24.485$

1 — *" given 2 sides & 1 included angle "*



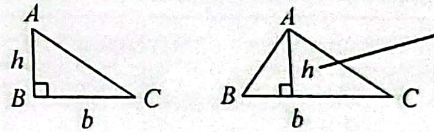
$AC^2 = 20^2 + 15^2 - 2(20)(15)\cos 65$

$AC = 19.27$

" Area of Triangle "

1 — $area\ \Delta ABC = \frac{1}{2}(20)(15)\sin 65 = 135.95$

1a

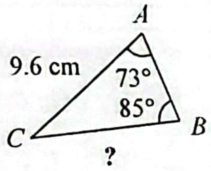


$area\ \Delta ABC = \frac{1}{2} \times b \times h$

shortest distance A to BC (perpendicular distance A to BC)

" Sine Rule "

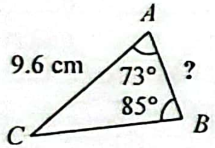
1a — *" given 1 side & 2 angles "*



$$\frac{BC}{\sin 73} = \frac{9.6}{\sin 85}$$

$BC = 9.216$

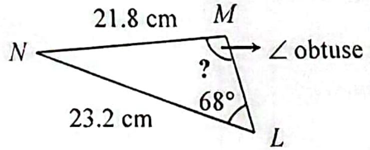
1b



- $\angle ACB = 180 - 73 - 85 = 22°$
- $\frac{AB}{\sin 22} = \frac{9.6}{\sin 85}$

  $AB = 3.610$

2a — *" given 2 sides & 1 non-included angle "*



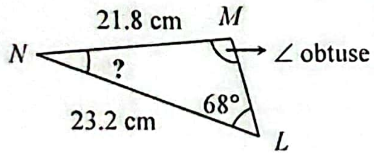
$$\frac{\sin \angle LMN}{23.2} = \frac{\sin 68}{21.8}$$

$\angle LMN = 80.65$ (acute)

$\angle LMN = 99.35$ (obtuse)

[*HINT* : ∠acute + ∠obtuse = 180°]

2b

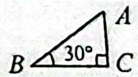


- $\frac{\sin \angle LMN}{23.2} = \frac{\sin 68}{21.8}$

  $\angle LMN = 80.65$ (acute)

  $\angle LMN = 99.35$ (obtuse)
- $\angle LNM = 180 - 68 - 99.35 = 12.65°$

" Ambiguous Case "



" only one ΔABC "

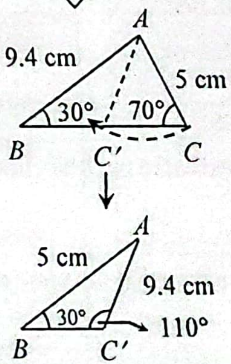
sketch new $\Delta ABC'$

~ AB maintain
~ AC maintain
~ $\angle ABC$ maintain

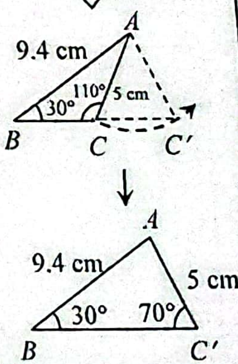
" HINT "

$\angle ACB + \angle AC'B = 180°$

1



2



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ONE PAGE NOTES [Coding Method]

" INDEX NUMBERS "

Additional notes

- h increased by 5 %
  $\Rightarrow h \times 105\ \%$
  $\Rightarrow h \times 1.05$
- k decreased by 25 %
  $\Rightarrow k \times 75\ \%$
  $\Rightarrow k \times 0.75$

$I_{21,16} = h$ and the price will increase at a constant rate of $k\%$ per year from the year 2021. Find $I_{26,16}$
$\Rightarrow I_{26,16} = h \times [\,(100 + k)\,\%\,]^5$

① *" Write information in Mathematical Representation & Vice-versa "* ②

- The price of an item in the year 2009 is RM 20
  $\Rightarrow Q_{09} = 20$
- The price index of an item in the year 2009 based on the year 2008 is 120
  $\Rightarrow I_{09,08} = 120$
- The price of an item has increased by 20 % from the year 2008 to the year 2009
  $\Rightarrow I_{09,08} = 120$
- The price of an item has decreased by 20 % from the year 2008 to the year 2009
  $\Rightarrow I_{09,08} = 80$
- The price of an item is unchanged from the year 2008 to the year 2009
  $\Rightarrow I_{09,08} = 100$

" Find Price "

△1 • Given : $I_{b,a} = k$ & Q_a @ Q_b $\Rightarrow \dfrac{Q_b}{Q_a} \times 100 = k$ @ $\dfrac{Q_b}{Q_a} = \dfrac{k}{100}$

△2 • Given : $I_{a,c}$, $I_{b,c}$ & Q_a @ Q_b $\Rightarrow \dfrac{I_{a,c}}{I_{b,c}} = \dfrac{Q_a}{Q_b}$ @ $\dfrac{I_{b,c}}{I_{a,c}} = \dfrac{Q_b}{Q_a}$

△3 • Given : $I_{c,a}$, $I_{c,b}$ & Q_a @ Q_b $\Rightarrow \dfrac{I_{c,a}}{I_{c,b}} = \dfrac{Q_b}{Q_a}$ @ $\dfrac{I_{c,b}}{I_{c,a}} = \dfrac{Q_a}{Q_b}$

△4 • Given : $I_{b,c}$, $I_{c,a}$ & Q_a @ Q_b $\Rightarrow \dfrac{I_{b,c}}{100} \times \dfrac{I_{c,a}}{100} = \dfrac{Q_b}{Q_a}$ ~ *continuously*

" Find Index "

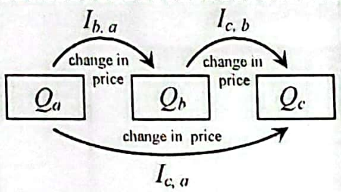
△1 • Given : Q_a & Q_b $\Rightarrow I_{b,a} = \dfrac{Q_b}{Q_a} \times 100$

[1] • Given : $I_{b,a}$ $\Rightarrow I_{a,b} = \dfrac{100}{I_{b,a}} \times 100$

[2] • Given : $I_{a,c}$ & $I_{b,c}$ $\Rightarrow I_{b,a} = \dfrac{I_{b,c}}{I_{a,c}} \times 100$

[3] • Given : $I_{c,a}$ & $I_{c,b}$ $\Rightarrow I_{b,a} = \dfrac{I_{c,a}}{I_{c,b}} \times 100$

[4] • Given : $I_{b,c}$ & $I_{c,a}$ $\Rightarrow I_{b,a} = \dfrac{I_{b,c} \times I_{c,a}}{100}$ ~ *case changes continuously*



- $I_{c,a} = \dfrac{I_{b,a} \times I_{c,b}}{100}$ (4)
- $I_{b,a} = \dfrac{I_{c,a}}{I_{c,b}} \times 100$ (3)
- $I_{c,b} = \dfrac{I_{c,a}}{I_{b,a}} \times 100$ (2)

☞ Given : $I_{b,a} = h$ & the price / index increased by $k\%$ from year b to c
$\Rightarrow I_{c,a} = h \times (100 + k)\,\%$ [4a]

☞ Given : $I_{b,a} = h$ & the price / index increased at the same rate from year b to c
$\Rightarrow I_{c,a} = h \times h\%$ [4b]

" Find Composite Index "

- Given : $I_{b,a}$ @ Q_a , Q_b & weightage (w)
  $\Rightarrow \bar{I}_{b,a} = \dfrac{\sum I_{b,a} w}{\sum w}$ ♡1 ♡2 ♡3

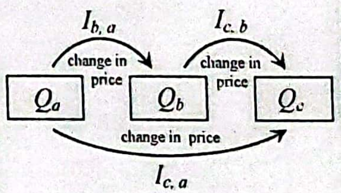
☞ weightage given in the form :
→ histogram, pai chart, ratio, %

☞ weightage is not given
→ assuming the value of the weightage are the same for each index number (1 : 1 : 1 : . . .)

- find corresponding price. △1

NOTE :
weightage → in % → $\sum w = 100\%$
weightage → in pai chart → $\sum w = 360°$

" find Price "



- $Q_b = Q_a \times (I_{b,a})\,\%$
  $Q_b = Q_a \times \dfrac{I_{c,a}}{I_{c,b}}$ (3)
- $Q_a = Q_b \div (I_{b,a})\,\%$
  $Q_a = Q_b \div \dfrac{I_{c,a}}{I_{c,b}}$
- $Q_c = Q_a \times (I_{b,a})\% \times (I_{c,b})\%$ (4)
  $Q_c = Q_a \times (I_{c,a})\,\%$
  $Q_c = Q_b \times (I_{c,b})\,\%$
  $Q_c = Q_b \times \dfrac{I_{c,a}}{I_{b,a}}$ (2)

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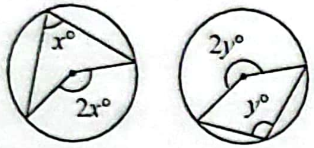
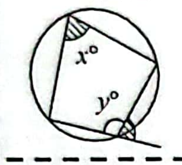
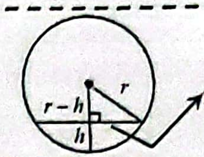
ONE PAGE NOTES
" CIRCULAR MEASURES "





$x + y = 180$

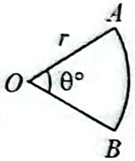


$\sqrt{r^2 - (r-h)^2}$

Recognise Cirlce

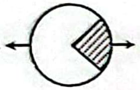
Circumference $= 2\pi r = \pi d$

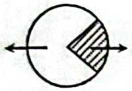
Area $= \pi r^2$

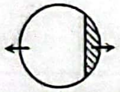


Arc length of AB, $S_{AB} = \dfrac{\theta}{360} \times 2\pi r$

Area of sector OAB, $A_{OAB} = \dfrac{\theta}{360} \times \pi r^2$







Unit Coversion

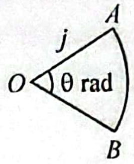
degrees $\xrightarrow{\times \frac{\pi}{180}}$ radians

radians $\xrightarrow{\times \frac{180}{\pi}}$ degrees

《 $180° = \pi$ 》

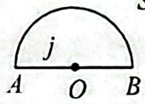
$30° = \frac{\pi}{6}$ rad	$90° = \frac{\pi}{2}$ rad
$45° = \frac{\pi}{4}$ rad	$270° = \frac{3\pi}{2}$ rad
$60° = \frac{\pi}{3}$ rad	$360° = 2\pi$ rad

Arc Length & Area of Sector



$S_{AB} = \theta r$

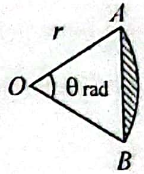
$A_{OAB} = \dfrac{1}{2}\theta r^2$



$S_{AB} = \pi r = \dfrac{1}{2}\pi d$

$A_{OAB} = \dfrac{1}{2}\pi r^2$

Length of Chord, Area Δ & Area of Segment



Chord AB $= 2r \sin\left(\dfrac{\theta}{2}\right)$

$Area_{\Delta OAB} = \dfrac{1}{2} r^2 \sin\theta$

$Area_{segment} = \dfrac{1}{2} r^2 (\theta - \sin\theta)$

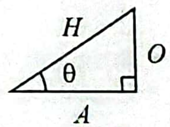
Pythagoras' Theorems & Trigonometric Ratio → Find : θ @ radius @ length of side

$H = \sqrt{O^2 + A^2}$

$O = \sqrt{H^2 - A^2}$

$A = \sqrt{H^2 - O^2}$

3	4	5	5	12	13
6	8	10	8	15	17
9	12	15	7	24	25



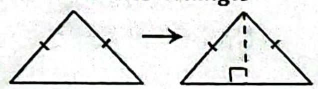
area $\Delta = \frac{1}{2} \times O \times A$

$\sin\theta = \dfrac{O}{H}$

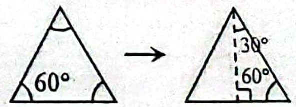
$\cos\theta = \dfrac{A}{H}$

$\tan\theta = \dfrac{O}{A}$

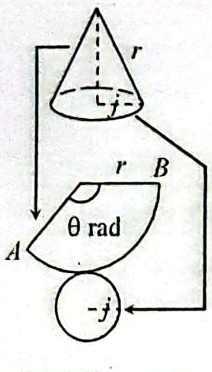
Isosceles Triangle



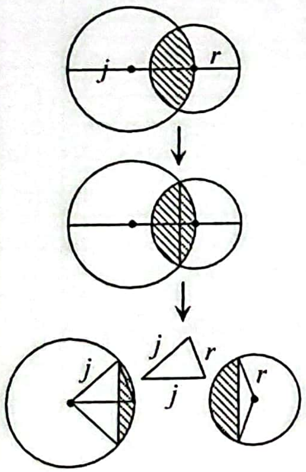
Equilateral Triangle

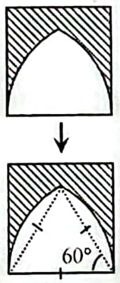


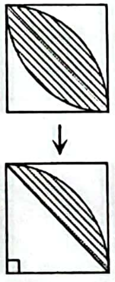
Cone & Layout



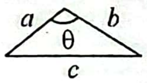
$S_{AB} = \theta r = 2\pi j$







Solution of Triangle

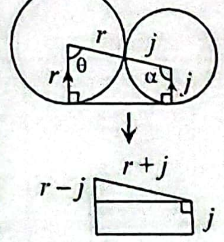


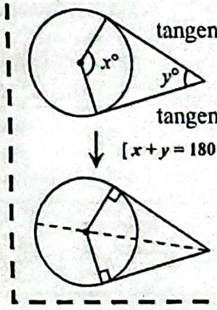
$\cos\theta = \dfrac{a^2 + b^2 - c^2}{2ab}$

area $\Delta = \frac{1}{2} ab \sin\theta$

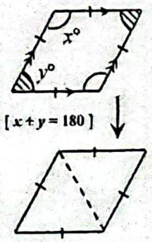
$c^2 = a^2 + b^2 - 2ab\cos\theta$

$\theta + \alpha = 180$





Rhombus



"Heron's formula" : area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, $s = \dfrac{a+b+c}{2}$ = semi perimeter

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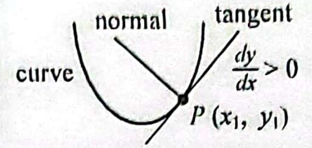
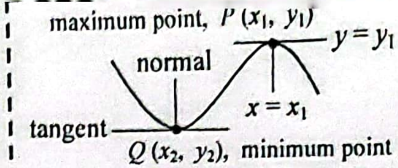
ONE PAGE NOTES

" DIFFERENTIATION"

gradient function $= \frac{dy}{dx} = f'(x)$

rate of change of radius $= \frac{dr}{dt}$





$P \rightarrow \frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

$Q \rightarrow \frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

LIMIT

- $\lim_{x \to a} f(x) = f(a)$
  ~ if $f(a) = \frac{0}{0}$ (undefine) → factor / rationalise
- if $0 < a < 1 \rightarrow \lim_{n \to \infty} (a)^n = 0$
- $\lim_{n \to \infty} \frac{1}{n} = 0$ @ $\lim_{n \to \infty} \frac{a}{n} = 0$
- if $\lim_{n \to \infty} f(x) = \frac{\infty}{\infty}$ ~ limit can't be obtained
  → divide each term in $f(x)$ by the highest power of x
- $\lim_{x \to a} f(x)$ exist $\rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
  → f is continuous at $x = a$

⟨⟨ APLICATION 1 ⟩⟩

EQUATION OF TANGENT / NORMAL

at a point (x_1, y_1) on a curve $y = f(x)$

(1) $\frac{dy}{dx}$ → m_1 (gradient / gradient of tangent)

(2) $x_1 \longleftrightarrow y_1$

(3) $y = m_1 x + c$ (equation of tangent)

(4) $m_1 \times m_2 = -1$ (m_2 = gradient of normal)

(5) $y = m_2 x + c$ (equation of normal)

⟨⟨ APLICATION 2 ⟩⟩

TURNING POINT / STATIONARY POINT

(minimum point / maximum point)

(1) $\frac{dy}{dx}$

turning point (x_1, y_1)

(2) $\frac{dy}{dx} = 0 \longrightarrow x_1 \longrightarrow y_1$

(3) $\frac{d^2y}{dx^2}$ ← $\frac{d^2y}{dx^2} > 0$ [minimum]

$\frac{d^2y}{dx^2} = 0$ [inflection] $\frac{d^2y}{dx^2} < 0$ [maximum]

⟨⟨ APLICATION 4 ⟩⟩

SMALL CHANGES

(1) $\frac{dy}{dx}$ additional informations

(2) $x_1 \longleftrightarrow y_1$

(3) determine δx @ δy

(4) $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

(5) $y_{new} = y_1 + \delta y$

(6) $\% \delta y = \frac{\delta y}{y_1} \times 100$

FIRST DERIVATIVE

" FIRST PRINCIPLES "

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

@

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

NOTES :

- tangent // x–axis → gradient, $\frac{dy}{dx}$, $m_1 = 0$
- tangent // a line → line : m_1
- tangent ⊥ a line → line : m_2
- normal // a line → line : m_2
- normal ⊥ a line → line : m_1

⟨⟨ APLICATION 3 ⟩⟩

RATES OF CHANGE

(1) $\frac{dy}{dx}$ addtional informations

(2) $x_1 \longleftrightarrow y_1$

(3) given $\frac{dx}{dt} \rightarrow \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

given $\frac{dy}{dt} \rightarrow \frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

FIRST DERIVATIVE

" FORMULAE 2 "

$y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$y(u)$ & $u(x)$
chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

FIRST DERIVATIVE

" FORMULAE 1 "

$y = f(x) \xrightarrow[\text{derivative}]{\text{first}} \frac{dy}{dx} = f'(x) \xrightarrow[\text{derivative}]{\text{second}} \frac{d^2y}{dx^2} = f''(x)$

$\frac{d}{dx}(k) = 0$

$\frac{d}{dx}(kx) = k$

$\frac{d}{dx}(kx^n) = (nk)x^{n-1}$

$\frac{d}{dx}\left(\frac{k}{x^n}\right) = \frac{-nk}{x^{n+1}}$

$\frac{d}{dx}[k(ax+b)^n] = (nk)(ax+b)^{n-1}(a)$

$\frac{d}{dx}\left(\frac{k}{(ax+b)^n}\right) = \frac{-nk(a)}{(ax+b)^{n+1}}$

EXAMPLES 1 :

$y = 3x^4 - \frac{x}{5} + 8$

$\rightarrow \frac{dy}{dx} = 12x^3 - \frac{1}{5}$

$\rightarrow \frac{d^2y}{dx^2} = 36x^2$

EXAMPLES 2 :

$f(x) = (3x-2)^5 + \frac{3}{x^2}$

$\rightarrow f'(x) = 5(3x-2)^4(3) - \frac{6}{x^3}$

$\rightarrow f''(x) = 60(3x-2)^3(3) + \frac{18}{x^4}$

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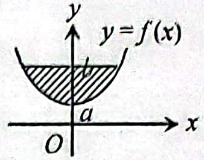
ONE PAGE NOTES
" INTEGRATION "

HINT :

$$\int \frac{x^3-2x}{x}\,dx = \int x^2-2\,dx$$

$$\int \frac{x^2-9}{x+3}\,dx = \int x-3\,dx$$

also applied to "DIFFERENTIATION"



volume of shaded area revolved 360° at x-axis

= volume of shaded area revolved 180° at x-axis

$$= \pi\int_a^b x^2\,dy$$

"INTEGRATION" REVERSE "DIFFERENTIATION"

$$\frac{d}{dx}(A) = B$$

b

$$\int B\,dx = A$$

$$y = A \text{ and } \frac{dy}{dx} = B$$

b

$$\int B\,dx = A$$

INTEGRATION " FORMULAE "

$$\int k\,dx = kx + c$$

$$\int kx^n\,dx = \frac{kx^{n+1}}{n+1} + c,\ n \neq -1$$

$$\int \frac{k}{x^n}\,dx = \frac{k}{x^{n-1}[-(n-1)]} + c,\ n \neq 1$$

$$\int k(ax+b)^n\,dx = \frac{k(ax+b)^{n+1}}{(n+1)(a)} + c,\ n \neq -1$$

$$\int \frac{k}{(ax+b)^n}\,dx = \frac{k}{(ax+b)^{n-1}[-(n-1)(a)]} + c,\ n \neq 1$$

DEFINITE INTEGRALS

$$\int_a^b f(x)\,dx = [\,F(x)\,]_a^b = F(b) - F(a)$$

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

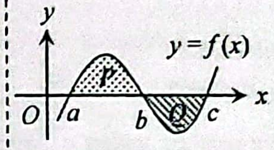
@

$$\int_a^b f(x)\,dx = k \leftrightarrow \int_b^a f(x)\,dx = -k$$

Note :

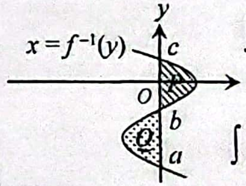
$$\int_a^a f(x)\,dx = 0$$

AREA



$$\int_a^b f(x)\,dx = h \Rightarrow \text{area } P = h$$

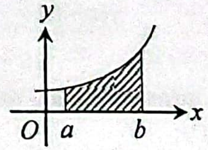
$$\int_b^c f(x)\,dx = k \Rightarrow \text{area } Q = |\,k\,|$$



$$\int_b^c f^{-1}(y)\,dy = h \Rightarrow \text{area } P = h$$

$$\int_a^b f^{-1}(y)\,dy = k \Rightarrow \text{area } Q = |\,k\,|$$

AREA and VOLUME

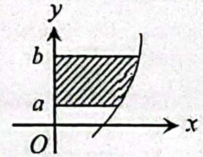


shaded area on x-axis

$$\int_a^b y\,dx$$

volume of shaded area revolved 360° at x-axis

$$\pi\int_a^b y^2\,dx$$

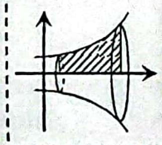


shaded area on y-axis

$$\int_a^b x\,dy$$

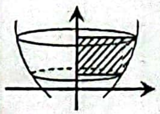
volume of shaded area revolved 360° at y-axis

$$\pi\int_a^b x^2\,dy$$



revolved

$[\times\frac{1}{4}]$ 90° 180° $[\times\frac{1}{2}]$



EXAMPLE 1 :

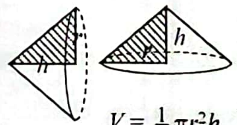
$$\int \frac{5x^3}{2} - \frac{4}{\sqrt{x}} + 8\,dx$$

$$= \frac{5x^4}{2(4)} - \frac{4}{x^{-\frac{1}{2}}(\frac{1}{2})} + 8x + c$$

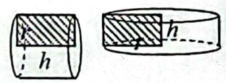
EXAMPLE 2 :

$$\int (1-x)^4 + \frac{2}{(3x-5)^3}\,dx$$

$$= \frac{(1-x)^5}{5(-1)} - \frac{2}{(3x-5)^2(-2)(3)} + c$$



$V = \frac{1}{3}\pi r^2 h$



$V = \pi r^2 h$

" PERMUTATION AND COMBINATION "

PERMUTATION → order of arrangement is important

~ *keywords* : the number of arrangements / codes / passwords / digit numbers . . .

- *multiplication rule*
  ~ If event A occur in r ways, and event B occur in s ways
  ⇒ *the number of ways* $= r \times s$
- *permutations of* n *different objects*
  ⇒ *the number of ways* $= n! = {}^nP_n$
- *permutations of* n *different objects, taking* r *objects each time*
  ⇒ *number of ways* $= {}^nP_r = \dfrac{n!}{(n-r)!}$; where $r \le n$
- *permutations for* n *objects involving identical objects*
  ⇒ *the number of ways* $= \dfrac{n!}{a!b!c!\ldots}$, where $a, b, c, \ldots$ are the number of identical objects for each type
- *circular permutations of* n *different objects* ~ round table, . . .
  (clockwise or anticlockwise arrangement are different)
  ⇒ *the number of ways* $= \dfrac{n!}{n} = \dfrac{n(n-1)!}{n} = (n-1)!$
- *circular permutations of* n *different objects taking* r *each time*
  ⇒ *the number of ways* $= \dfrac{{}^nP_r}{r}$, where $r \le n$
- *arrangements of* n *different objects to form a circular ring* ~ bracelet, necklace, ...
  (do not involve clockwise or anticlockwise, because both are the same)
  ⇒ *the number of ways* $= \dfrac{n!}{2n} = \dfrac{n(n-1)!}{2n} = \dfrac{(n-1)!}{2}$
- *arrangements of* n *different objects taking* r *each time, to form a circular ring*
  ⇒ *the number of ways* $= \dfrac{{}^nP_r}{2r}$, where $r \le n$

Notes :

- *even number* → last digit must be even
- *odd number* → last digit must be odd
- *divisible by* 5 → last digit must be 0, 5
- *divisible by* 2 → last digit must be even

COMBINATION → order of arrangement is not important

~ *keywords* : the number of ways / selections / committees / teams / groups / colours / parcels / hampers . . .

- *combinations of* r *objects chosen from* n *different objects*
  (without considering the positions or arrangements)
  ⇒ *the number of ways* $= {}^nC_r = \dfrac{n!}{(n-r)!\,r!} = \dfrac{{}^nP_r}{r!}$, where $r \le n$.

Notes :

- less than ($<$)
- more than, greater than ($>$)
- at most, not more than, the maximum ($\le$)
- at least, not less than, the minimum ($\ge$)

- $n! = n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)\ \ldots\ (5)(4)(3)(2)(1)$
  $= n(n-1)!$
  $= n(n-1)(n-2)!$
  $= n(n-1)(n-2)(n-3)!$
  $= n(n-1)(n-2)(n-3)(n-4)!$
  ↓
  $n! = {}^nP_n = {}^nP_{n-1}$ ⇒ example : $8! = {}^8P_8 = {}^8P_7$

  example
  $6! = 6(5)(4)(3)(2)(1)$
  $= 6(5)!$
  $= 6(5)(4)!$
  $= 6(5)(4)(3)!$

- ${}^nP_r = \dfrac{n!}{(n-r)!} = n(n-1)(n-2)\ \ldots\ (n-r-1)(n-r)(n-r+1)$ → ${}^9P_4 = 9(8)(7)(6)$

  ⇒ ${}^nP_{r-1} = \dfrac{n!}{(n-r+1)!}$; ${}^nP_{r-2} = \dfrac{n!}{(n-r+2)!}$; ${}^{n-1}P_{r-1} = \dfrac{(n-1)!}{(n-r)!}$; . . .

  ⇒ ${}^nP_{r+1} = \dfrac{n!}{(n-r-1)!}$; ${}^nP_{r+2} = \dfrac{n!}{(n-r-2)!}$; ${}^{n-1}P_{r+1} = \dfrac{(n-1)!}{(n-r-2)!}$; . . .

- ${}^nC_r = \dfrac{n!}{(n-r)!\,r!} = {}^nC_{n-r}$ ⇒ *example* : ${}^6C_2 = {}^6C_4$ ($2+4=6$) ; ${}^{10}C_3 = {}^{10}C_7$ ($3+7=10$) . . .

- ${}^nP_1 = {}^nC_1 = {}^nC_{n-1} = n$

- $0! = 1! = {}^nP_0 = {}^nC_0 = {}^nC_n = 1$ ⇒ ${}^nP_r > 1 \sim r > 0$ *and* ${}^nC_r > 1 \sim 0 < r < n$

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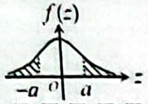
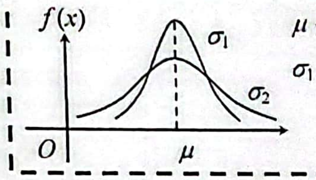
ONE PAGE NOTES [Coding Method]
"PROBABOLITY DISTRIBUTIONS"

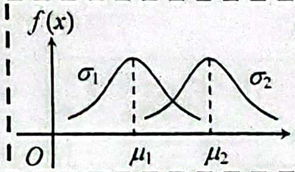


$P(|z| < a) = P(-a < z < a) = 1 - 2P(z > a)$

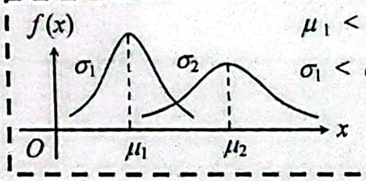
$P(|z| > a) = P(z < -a) + P(z > a) = 2P(z > a)$



$\mu_1 = \mu_2$, $\sigma_1 < \sigma_2$



$\mu_1 < \mu_2$, $\sigma_1 = \sigma_2$



$\mu_1 < \mu_2$, $\sigma_1 < \sigma_2$

BINOMIAL DISTRIBUTION

$X \sim B(n, p)$ → X discrete random variable ; $X = 0, 1, 2, 3, 4, 5, \ldots, n$

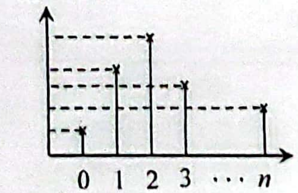
$P(X = r) = {}^nC_r \; p^r \; q^{n-r}$
- ~ r = number of success ($r = 0, 1, 2, 3, \ldots, n$)
- ~ n = number of trials
- ~ p = probability of success ($0 < p < 1$)
- ~ q = probability of failure ($q = 1 - p$ @ $p + q = 1$)

(1)

percentage ÷ 100 ↓ probability ; probability × 100 ↑ percentage

${}^nC_0 = 1$, ${}^nC_n = 1$, $a^0 = 1$ & ${}^nC_1 = n$

P (X = x)



- ➲ ${}^nC_r = \frac{n!}{(n-r)!\,r!}$ ⇒ ${}^nC_r = {}^nC_{n-r}$
- ➲ $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$
  $= n(n-1)! = n(n-1)(n-2)! = \ldots$
  $8! = 8(7!) = 8(7)(6!) = \ldots$

$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + \ldots\ldots\ldots\ldots + P(X = n) = 1$

(2) ex $P(X < 2) = P(X = 0) + P(X = 1)$

(3) ex $P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

- mean @ expected value of X, $\mu = np$
- variance of X, $\sigma^2 = npq$
- standard deviation of X, $\sigma = \sqrt{npq}$

(4)

Notes :
- ➲ less / small than (<)
- ➲ at most, not more than, maximum (≤)
- ➲ more / greater than (>)
- ➲ at least, not less than, minimum (≥)

Find $n(A)$ **&** $n(S)$ △2
[first, find the value of probabity]

$P(A) = \frac{n(A)}{n(S)}$

calculator ***fx*-570MS**

[mode] ; [mode] ; [1] ;
[shift] ; [3]

$R \sim >, \geq$ & $P \sim <, \leq$

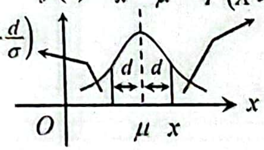
NORMAL DISTRIBUTION

[1σ from $\bar{x}$ = $-1 < z < 1$; 2σ from $\bar{x}$ = $-2 < z < 2$; . . .]

$X \sim N(\mu, \sigma^2)$ → X continuous random variable ; $X < a$, $X > a$, $a < X < b$, . . .

Normal distribution, $X \sim N(\mu, \sigma^2)$

Standard normal distribution, $Z \sim N(0, 1)$

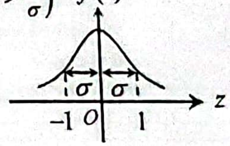


$P\left(Z < -\frac{d}{\sigma}\right)$

$P(X > x) = P\left(Z > \frac{x-\mu}{\sigma}\right) = \left(Z > \frac{d}{\sigma}\right)$

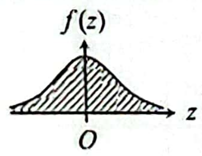
$z = \frac{x - \mu}{\sigma}$

$d = x - \mu$

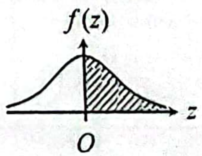


$\mu = 0$
$\sigma^2 = 1$
$\sigma = 1$

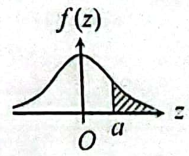
Find Probability [given x-score, convert to z-score, find the probability value] △1



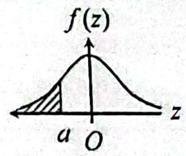
$P(-\infty < z < \infty) = 1$



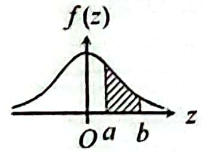
$P(z > 0) = 0.5$



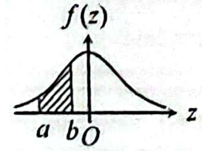
$P(z > a) = R(a)$



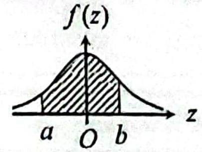
$P(z < a) = P(a)$



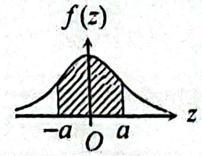
$P(a < Z < b)$
$= P(Z > a) - P(Z > b)$



$P(a \leq Z \leq b)$
$= P(Z < b) - P(Z < a)$



$P(a < Z \leq b)$
$= 1 - P(Z < a) - P(Z > b)$



$P(-a \leq Z < a)$
$= 1 - 2P(Z > a)$

Find ***x*-score** [get the probability value, find the corresponding z-score, solve] △3

《 **EXAMPLE** 》

- $P(x > k) = 0.4$

  $P\left(z > \frac{k-\mu}{\sigma}\right) = 0.4$

  ~ $P(z > \underline{0.253}) = 0.4$ [from table]

  $\frac{k-\mu}{\sigma} = 0.253$

- $P(x > k) = 0.87$

  $P\left(z < \frac{k-\mu}{\sigma}\right) = 0.13$

  ~ $P(z > \underline{1.127}) = 0.13$ [from table]

  $\frac{k-\mu}{\sigma} = -1.127$

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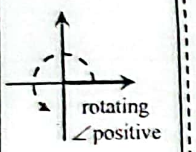
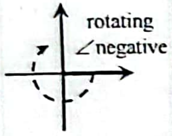
ONE PAGE NOTES

" TRIGONOMETRIC FUNCTIONS "

ROTAING OF ANGLE / RELATION BETWEEN ANGLES / 3 BASIC TRIGONOMERICS RATIOS / PYTHAGORAS THEOREM





Quadrant II ~ obtuse ($90° < \theta < 180°$)
- $\sin\theta \to +$
- $\cos\theta \to -$
- $\tan\theta \to -$

Quadrant I ~ acute ($0° < \theta < 90°$)
- $\sin\theta \to +$
- $\cos\theta \to +$
- $\tan\theta \to +$

Quadrant III ~ reflex ($180° < \theta < 270°$)
- $\sin\theta \to -$
- $\cos\theta \to -$
- $\tan\theta \to +$

Quadrant IV ~ reflex ($270° < \theta < 360°$)
- $\sin\theta \to -$
- $\cos\theta \to +$
- $\tan\theta \to -$

sin → + (I, II); − (III, IV)

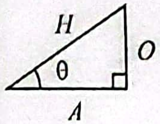
cos → + (I, IV); − (II, III)

tan → + (I, III); − (II, IV)

I → II: 180 − I; II → I: 180 − II

I → III: 180 + I; III → I: III − 180

I → IV: 360 − I; IV → I: 360 − IV



$H = \sqrt{O^2 + A^2}$

$O = \sqrt{H^2 - A^2}$

$A = \sqrt{H^2 - O^2}$

$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

3	4	5
6	8	10
9	12	15

5	12	13
7	24	25
8	15	17

SPECIAL ANGLE

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt2} = \frac{\sqrt2}{2}$	$\frac{\sqrt3}{2}$
cos	$\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2} = \frac{\sqrt2}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt3} = \frac{\sqrt3}{3}$	1	$\sqrt3$

COMPLEMENTARY ANGLE FORMULAE

$\sec x = \frac{1}{\cos x} \Rightarrow \cos x = \frac{1}{\sec x}$

$\operatorname{cosec} x = \frac{1}{\sin x} \Rightarrow \sin x = \frac{1}{\operatorname{cosec} x}$

$\cot x = \frac{1}{\tan x} \Rightarrow \tan x = \frac{1}{\cot x}$

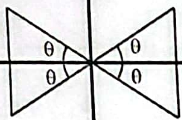
$\tan x = \frac{\sin x}{\cos x}$ / $\cot x = \frac{\cos x}{\sin x}$

- $\operatorname{cosec}(180° - \theta) = \frac{1}{\sin\theta}$
- $\sec(180° - \theta) = -\frac{1}{\cos\theta}$
- $\cot(180° - \theta) = -\frac{1}{\tan\theta}$

Quadrant II
- $\sin(180° - \theta) = \sin\theta$
- $\cos(180° - \theta) = -\cos\theta$
- $\tan(180° - \theta) = -\tan\theta$

Quadrant I
- $\sin\theta$
- $\cos\theta$
- $\tan\theta$

trigonometric ratios for angles between 0° and 360°



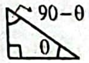
- $\operatorname{cosec}(180° + \theta) = -\frac{1}{\sin\theta}$
- $\sec(180° + \theta) = -\frac{1}{\cos\theta}$
- $\cot(180° + \theta) = \frac{1}{\tan\theta}$

Quadrant III
- $\sin(180° + \theta) = -\sin\theta$
- $\cos(180° + \theta) = -\cos\theta$
- $\tan(180° + \theta) = \tan\theta$

Quadrant IV
- $\sin(360° - \theta) = -\sin\theta$
- $\cos(360° - \theta) = \cos\theta$
- $\tan(360° - \theta) = -\tan\theta$

- $\operatorname{cosec}(360° - \theta) = -\frac{1}{\sin\theta}$
- $\sec(360° - \theta) = \frac{1}{\cos\theta}$
- $\cot(360° - \theta) = -\frac{1}{\tan\theta}$

For any



- $\sin(90 - \theta) = \cos\theta$
- $\cos(90 - \theta) = \sin\theta$
- $\tan(90 - \theta) = \cot\theta$
- $\operatorname{cosec}(90 - \theta) = \sec\theta$
- $\sec(90 - \theta) = \operatorname{cosec}\theta$
- $\cot(90 - \theta) = \tan\theta$

UNIT CIRCLE

- $\sin\theta =$ *y-coordinate*
- $\cos\theta =$ *x-coordinate*
- $\tan\theta = \frac{y\text{-coordinate}}{x\text{-coordinate}} = \frac{\sin\theta}{\cos\theta}$

NEGATIVE ANGLES → $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$; $\tan(-\theta) = -\tan\theta$; $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$; $\sec(-\theta) = \sec\theta$; $\cot(-\theta) = -\cot\theta$

BASIC IDENTITIES

- $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$
  $\Rightarrow \sin^2 x = 1 - \cos^2 x$
- $\sec^2 x = 1 + \tan^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$
- $\operatorname{cosec}^2 x = 1 + \cot^2 x \Rightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$

DOUBLE ANGLE FORMULAE / HALF ANGLE

- $\cos 2A = \cos^2 A - \sin^2 A \Rightarrow \cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$
  $= 2\cos^2 A - 1$; $= 2\cos^2\frac{A}{2} - 1$
  $= 1 - 2\sin^2 A$; $= 1 - 2\sin^2\frac{A}{2}$

ADDITION FORMULAE

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B$ m $\sin A\sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \text{ m } \tan A\tan B} \to \cot(A \pm B) = \frac{1 \text{ m } \tan A\tan B}{\tan A \pm \tan B}$

- $\sin 2A = 2\sin A\cos A \Rightarrow \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
- $\tan 2A = \frac{2\tan A}{1 - \tan^2 A} \Rightarrow \tan A = \frac{2\tan\frac{A}{2}}{1 - \tan^2\frac{A}{2}}$
- $\cot 2A = \frac{1 - \tan^2 A}{2\tan A} \Rightarrow \cot A = \frac{1 - \tan^2\frac{A}{2}}{2\tan\frac{A}{2}}$

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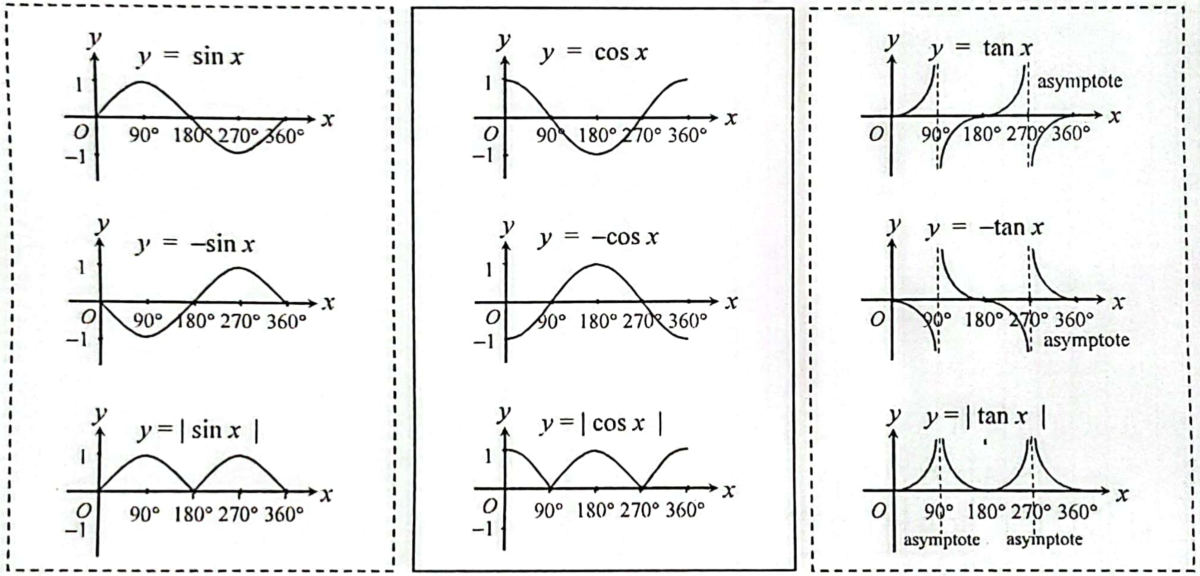
GRAPH → $y = a \sin bx$ @ $y = a \cos bx$ @ $y = a \tan bx$; where $a = 1$, $b = 1$

- $\sin x° = 0 \Rightarrow x = 0, 180, 360$
- $\sin x° = 1 \Rightarrow x = 90$
- $\sin x° = -1 \Rightarrow x = 270$

- $\cos x° = 0 \Rightarrow x = 90, 270$
- $\cos x° = 1 \Rightarrow x = 0, 360$
- $\cos x° = -1 \Rightarrow x = 180$

- $\tan x° = 0 \Rightarrow x = 0, 180, 360$
- $\tan x° = \infty \Rightarrow x = 90, 270$



In general :

	$y = a \sin bx + c$	$y = a \cos bx + c$	$y = a \tan bx + c$
amplitude	$\lvert a \rvert = \dfrac{\text{maximum-}y - \text{minimum-}y}{2}$		no amplitude
period	$\dfrac{360°}{b}$		$\dfrac{180°}{b}$
b	$\dfrac{360°}{\text{period}}$		$\dfrac{180°}{\text{period}}$
c	$\dfrac{\text{maximum-}y + \text{minimum-}y}{2}$		movement of the origin point up or down on the y-axis

~ a changes → the maximum and minimum value change
~ b changes → the graph will be compressed or expanded
~ c changes → the graph will be move vertically up or down

Notes :

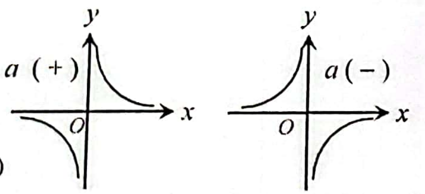
- *period* = the angle for one circle @ a graph shape repeating
- sketch graph : *class interval size* $= \dfrac{\pi}{2b} = \dfrac{90°}{b}$

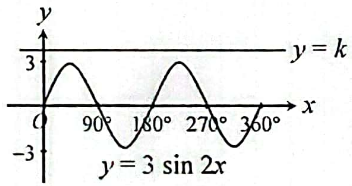
  draw graph : *class interval size* $= \dfrac{\pi}{4b} = \dfrac{45°}{b}$, for "sin" and "cos"

  class interval size $= \dfrac{\pi}{8b} = \dfrac{45°}{2b}$, for "tan"

$y = \dfrac{a}{x} \Rightarrow$

(reciprocal function)





number of possible solutions :

- if $3 \sin 2x = k$ has no solution → $k < -3$ and $k > 3$
- if $3 \sin 2x = k$ has 2 solutions → $k = -3$ and $k = 3$
- if $3 \sin 2x = k$ has 4 solutions → $-3 < k < 3$ and $k \neq 0$
- if $3 \sin 2x = k$ has 5 solutions → $k = 0$

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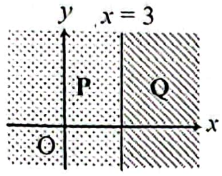
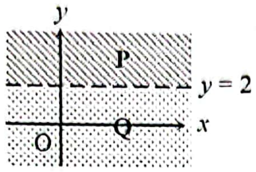
ONE PAGE NOTES
"LINEAR PROGRAMMING"

NOTE :



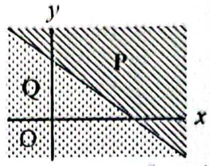
region P ~ $x \leq 3$

region Q ~ $x \geq 3$



region P ~ $y \geq 2$

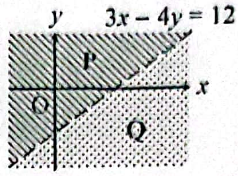
region Q ~ $x \leq 2$



$x + 2y = 4$

region P ~ $x + 2y \geq 4$

region Q ~ $x + 2y \leq 4$



region P ~ $3x - 2y \leq 12$

region Q ~ $3x - 2y \geq 12$

Steps to solve linear programming problems using graphical method :

(i) State the two variables involved ~ x , y

(ii) Write down a system of linear inequalities that satisfies all the constraints.

- at least, not less than, the minimum → $\geq$
- at most, not more than, the maximum → $\leq$
- less than → $<$
- more than → $>$

Examples :	
y is more than x	$\Rightarrow$ $y > x$
y is less than x	$\Rightarrow$ $y < x$
y is not more than x	$\Rightarrow$ $y \leq x$
y is not less than x	$\Rightarrow$ $y \geq x$
y is at most k times of x	$\Rightarrow$ $y \leq kx$
y is at least k times of x	$\Rightarrow$ $y \geq kx$
the maximum of y is k	$\Rightarrow$ $y \leq k$
the minimum of x is k	$\Rightarrow$ $x \geq k$
the total of x and y is at most k	$\Rightarrow$ $x + y \leq k$
y exceed x by at least k	$\Rightarrow$ $y - x \geq k$
y is at most k % that of x	$\Rightarrow$ $y \leq \frac{k}{100}x$
the ratio of y to x is at least 1 : 3	$\Rightarrow$ $y : x \geq 1 : 3$

(iii) Using the given scales for x and y axes, draw and shade the region R which satisfies all the inequalities.

(iv) Form an optimum function, $k = ax + by$.

(v) Select a suitable value of n, draw the straight line with na as the y- intercept and nb as the x-intercept, where $n = 1, 2, 3, \ldots$

Example :

× 2 × 5

$k = 5x + 8y$

$\Rightarrow$ x-intercept = 8	$\Rightarrow$ x-intercept = 16	$\Rightarrow$ x-intercept = 40
$\Rightarrow$ y-intercept = 5	$\Rightarrow$ y-intercept = 10	$\Rightarrow$ y-intercept = 25
$\therefore$ $5x + 8y = 40$	$\therefore$ $5x + 8y = 80$	$\therefore$ $5x + 8y = 200$

(vi) With the use of a ruler and set square, slide the line towards the region R to obtain the optimum point for the required optimum function.

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NOTE :

$$s \underset{\int v\,dt}{\overset{\frac{ds}{dt}}{\rightleftharpoons}} v \underset{\int a\,dt}{\overset{\frac{ds}{dt}}{\rightleftharpoons}} a$$

$s = 0$

- Returns/ passes through point O/ initial point.

$s > 0$
On the right of O.

$s < 0$
On the left of O.

$v = 0$

- Stop/ At rest/ Stationary.
- Reverse its direction of motion/ Turns to left/ right/ opposite direction.
- Maximum/ Minimum displacement.

$v > 0$
Moves to right.

$v < 0$
Moves to left.

$a = 0$

- Maximum/ Minimum velocity.
- Constant velocity/ speed.

$a > 0$
Velocity is increasing

$a < 0$
Velocity is decreasing

IMPLICATIONS :

- initial position (s), initial velocity (v), initial acceleration (a), passes O for the first time $\rightarrow t = 0$.
- P and Q meet $\rightarrow s_P = s_Q$.
- not return to O (s), not stop (v) $\rightarrow$ $b^2 - 4ac < 0$ (for function in quadratic form)
- the distance travelled during the n seconds / in the n seconds = $\int_{n-1}^{n} v \; dt = |s_n - s_{n-1}|$
- the total distance travelled by the particle in the first n seconds,

  (i) if $v \neq 0$, the total distance = $\int_0^n v \; dt = |s_n - s_0|$

  (ii) if $v = 0$ when $t = m$ where $0 \leq m \leq n$, the total distance = $\left|\int_0^m v \; dt\right| + \left|\int_m^n v \; dt\right|$

  # first 5 second → $t = 0$ until $t = 5$
  # the fifth second → $t = 4$ until $t = 5$

- average speed = $\dfrac{\text{total distance travelled}}{\text{total time taken}}$

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